

# OPTIMAL INVENTORY REDISTRIBUTION INTEGRATED WITH LATERAL TRANSSHIPMENTS AND EMERGENCY ORDERS

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## ABSTRACT

Inventory redistribution and emergency replenishment in supply chains have long been studied in various contexts. From a strategic perspective, lateral transshipments can be beneficial for a variety of general supply chains to cope with the mismatch between periodic realized demands and inventory levels. Similarly, the emergency replenishment from supply sources has also been used as a tool to mitigate the demand uncertainty. In this paper, we consider the integration of instantaneous transshipments and emergency shipments in a multi-location inventory system consisting of a distribution center, and multiple retailers. A mathematical optimization model is formulated first, and then an approach based on a Stochastic Genetic Algorithm (SGA) is presented to solve the model. The objective of the model is to determine optimal replenishment policies for the multi-location inventory system. A coarse-to-fine procedure is employed to reduce the computational time. Finally, computational results are given to show the validity of the presented approach.

## KEY WORDS

Inventory, Transshipment, Emergency Orders, and Genetic Algorithm

## 1. Introduction

Multi-location inventory systems have been investigated for several decades. The modeling approach of such systems is different from classical inventory models in that it aims to find inventory policy that is optimal for the entire system of multiple locations. As part of the research field, the investigation of N-location models with lateral transshipment is considered as an important problem for both inventory theory and inventory practice. However, the multi-location models are generally complex, difficult to solve, and also intractable. The complexity of the model is increased substantially when the model is formulated for capturing real-life situations such as stochastic behavior, or considering more dimensions of

inventory activities. The model cannot be often solved using traditional optimization methods.

The structure of multi-locations results in a system with multi-direction cooperation. Earlier studies began with the idea of centralizing the control of such systems. A centralized model with the concept of inventory pooling was first presented by Eppen [1]. The model was later extended in several studies to handle additional cost parameters and constraints. Tagaras [2], [3] considered the case of transshipments in a complete pooling where the retailers' cost parameters are identical. Robinson [4] examined a general case of multiple locations but with different cost structures. Herer [5] proposed multi-location transshipment models to solve for optimal policies using the network flow representation and Infinitesimal Perturbation Analysis (IPA) technique. Recently, Luo and Wang [6] applied Genetic Algorithm (GA) for determining an initial order quantity and the transshipment scheme in a complex multiple retailers network with different demand and inventory parameters.

*Emergency Orders* is one of the widely known options that has been used in a similar manner to the transshipment. Stocking location can request for another replenishment after a regular replenishment. However, the retailer may need to pay a higher purchasing cost for the replenishment in a very short lead-time. Tagaras and Vlachos [7], and Teunter and Vlachos [8] have studied systems with the emergency replenishment, and proposed periodic inventory models with two supply modes that can be used to determine the order-up-to replenishment parameters and approximate costs.

Most existing works on transshipment systems normally assumed that (1) lateral transshipments within a group of retailers are preferred rather than seeking products from external suppliers or using other options such as the emergency order system [5], [9], [10], (2) facilities are located closely in the area and they have potential to implement all transshipment activities (i.e., providing extra transportation and storage), and (3) locations are not completely independent and are willing to share both

information and inventories. These assumptions are quite conflicting to the general practice in commercial supply chains. The facilities may be owned by different companies who have joined the supply chain. Some of the locations may be located far from each others, and have not enough capabilities. Furthermore, the commercial network can be designed to transfer products efficiently either on the predetermined routes or in the lateral groups. Thus, we can see that only the lateral transshipment system may not provide the best advantage to commercial supply chains.

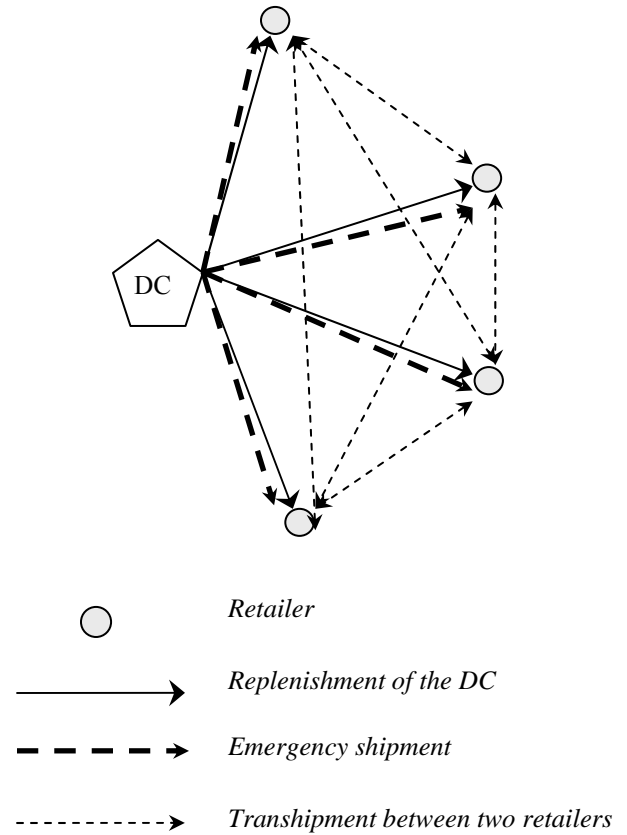
In this paper, we propose a strategic viewpoint of integrating retailers' lateral transshipments and emergency replenishments into the multi-location inventory system. We adopted the idea of adding an artificial distribution center in the network to integrate effectively the emergency ordering with the lateral transshipment. A mathematical model representing the system is formulated to determine the optimal order-up-to replenishment levels for retailers. The Stochastic Genetic Algorithm (SGA) approach is applied to the model such that the nature of stochastic demand is effectively handled. A Coarse-To-Fine approach is also applied to assist in the GA model in order to increase accuracy as well as reduce the computation time.

The remainder of this paper is organized as follows. The following section is devoted to describe the problem, the configuration of a distribution network under study, modeling assumptions, and model formulation. Solution approach is then discussed in Section 3. In Section 4, we conduct numerical experiments to show the validity of our approach. A final conclusion is made in Section 5.

## 2. Optimization Model

### 2.1 Modeling assumptions

Consider a distribution network of one distribution center (DC) serving for N retailer locations. A distribution center is considered as a supply node that replenishes the amount of a single product to retailers. Inventory level at each retailer is reviewed periodically and regular replenishments can be made after the retailers place their order requesting regular replenishments. Within the period, demand is observed at each location. Retailers, in case of shortage or imminent stock-out, will be able to seek additional inventory through the lateral transshipment system and/or the emergency ordering system. A distribution network with complete inventory pooling described above can be structured as shown in Figure 1.



**Figure 1:** A Distribution Network Concerned

The following assumptions are set for operations of all retailers:

- A distribution center has sufficient capacity to respond to any request from retailers.
- Retailers face independent demand distribution, which is stationary and known for a period.
- Retailers operate with different cost structure and different inventory parameters (i.e., holding costs, penalty costs, procurement costs, demands)
- Retailers employ an order-up-to inventory policy
- Both lateral transshipments and emergency replenishments can be carried out within the period after the realization of demands at retailers.
- Retailers cannot request more than one additional shipment in the same period.
- Unsatisfied demand is backlogged and will be satisfied using the regular replenishment at the beginning of the next period.

## 2.2 Model Formulation

### Notation

The following parameters are used for modeling formulation:

*Cost and Inventory Parameters:*

$i, I$  = index and set of retailers where  $i = 0$  implies the DC;

$d_i$  = observed demand of retailer  $i$ ;  $D = \{d_i\}$ ;

$c_{ij}$  = unit cost of lateral transshipment from retailer  $i$  to retailer  $j$ ;

$h_i$  = unit cost of holding inventory per period at retailer  $i$ ;

$p_i$  = unit cost of shortage backlogged per period at retailer  $i$ ;

*Decision Variables:*

$S_i$  = order-up-to level at retailer  $i$ ;  $S = \{S_i\}$ ;

$x_{ij}$  = the amount of the product transshipped from retailer  $i$  to retailer  $j$ ;  $X = \{x_{ij}\}$ ;

$I_i^+$  = net surplus at retailer  $i$  after transshipments and demand satisfaction;

$I_i^-$  = net shortage at retailer  $i$  after transshipments and demand satisfaction

### Objective function and constraints

The problem will be described as strategic inventory planning over a long planning horizon, which aims to determine an optimal replenishment policy for retailers under the network configuration of complete inventory pooling with unlimited supply. The objective of the problem is to minimize the total cost of the integrated multi-location system. We follow the concept of model formulation as discussed in Herer [5], [9]. The similar concept has also been simplified and used in the design of lateral transshipment networks [11]. As we integrated the emergency replenishment system into the previous model, we need to deal with the amount of product that flows instantaneously from a DC to retailers. An artificial retailer is added into the retailer group. This dummy retailer represents the function of DC, but acting like a retailer who can exchange a lateral transshipment quantity with any retailer at a cost of emergency replenishment (Figure 2). Thus, one-directional links from the artificial retailer to any retailer in the group can represent emergency orders. The model in Herer [5] and the simplified version of model in Lein [11] are still valid

here. We then formulate the multi-location inventory system which integrates lateral transshipment and emergency orders problem as follows:

$$\text{Min}_x Z(X|S,D) = \sum_{ij} c_{ij} x_{ij} + \sum_i (h_i I_i^+ + p_i I_i^-) \quad (1)$$

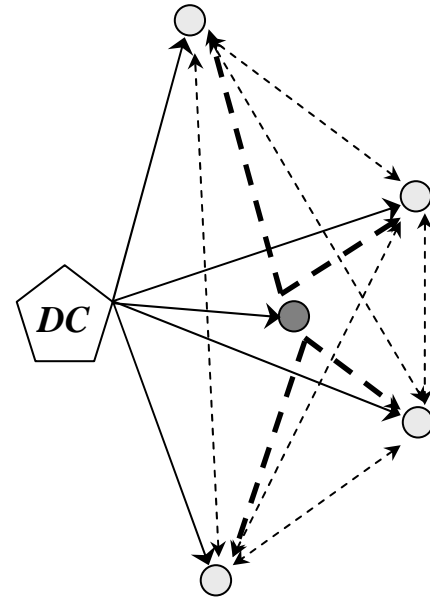
subject to

$$\sum_j x_{ij} - \sum_k x_{ki} + I_i^+ - I_i^- = S_i - d_i \quad \forall i \quad (2)$$

$$\sum_j x_{ij} + I_i^+ \leq S_i \quad \forall i \quad (3)$$

$$x_{ij} \geq 0 \quad \forall i, j \quad (4)$$

$$I_i^+, I_i^- \geq 0 \quad \forall i \quad (5)$$



● *Artificial retailer representing the DC*

**Figure 2:** An Artificial Retailer in The Multi-Location Inventory System

The objective function (1),  $\text{Min}_x Z(X | S,D)$ , minimizes the total cost of transshipment costs, emergency ordering costs, inventory holding and penalty costs. The model determines transshipment quantities  $X_{ij}$  based on the calculation of the total throughput of the system while assuming that the demand pattern ( $d_i$ ) and replenishment policy ( $S_i$ ) are given. Constraint (2) is the control equation to guarantee that demand will be satisfied at each retailer location. Constraint (3) limits the amount of transshipment that could occur in a period. Constraint (4) and (5) represent the non-negativity of the decision variables.

### 3. A Stochastic Genetic Algorithm

#### 3.1 Concept

We intend to solve the model for optimal order-up-to level at each retailer under a stochastic behavior. Note that Herer [5], [9] has proposed an Infinitesimal perturbation Analysis (IPA) procedure for solving his model, and Lein [11] also followed the same procedure for his simplified model in the design of transshipment networks. A different solution methodology, using a Stochastic Genetic Algorithm (SGA), is suggested here to solve for optimal values of this integrated model. The  $\text{Min}_X Z(X | S, D)$  model is now converted into an expected value model preparing for the GA optimization under a stochastic behavior as follows:

$$\text{Min}_S E_D[Z^*(X^* | S, D)] \quad (6)$$

where,  $[Z^*(X^* | S, D)] = \text{Min}_X Z(X | S, D)$   
 $Z^*$  = minimum of the total supply cost from the objective function (1)  
 $X^*$  = optimal solutions of the model with objective function (1)

The objective function (6),  $\text{Min}_S E_D[Z^*(X^* | S, D)]$ , would minimize the total expected cost ( $E_D$ ) of the supply chain based on the optimal values of  $X_{ij}$  that is calculated from the objective function (1) with respect to the set of constraints. Stochastic nature of retailers' demand will be materialized by randomly generating a set of demand patterns. At this stage, the demand values are known, and different  $S_i$  values can also be known while GA chromosomes are encoded, the previous formulation in Section 2.2 is already reduced into a simple linear programming (LP) model. After solving LP for multiple sets of optimal  $Z^*$  and  $X^*$  that associated with the known  $d_j$  and  $S_i$  values for a specific chromosome, an average of  $Z^*$  values will represent the fitness value (expected total cost of the system,  $E_D$ ) for that chromosome. In the optimization process using the GA by changing  $S_i$  values, the objective function (6) would finally converge to the minimum of the expected total supply chain cost.

Considering that the linear programming sub-model is used to evaluate the fitness value of chromosomes, the runtime of the LP and the GA routine definitely depends on the number of locations, the number of demand patterns generating for a chromosome, the number of chromosomes in the population, and the number of total generations of the GA. It is important to generate enough different demand patterns so that the model can mimic the stochastic behavior approximately. It is also important to set appropriate numbers of populations and generations so that the GA can generate more possible solutions throughout the search space. We introduced a Coarse-to-

Fine approach to assist the GA model in order to increase accuracy as well as reduce the computational time. As the generation of the GA goes, the model investigates more demand patterns while the number of chromosomes in the population is decreased.

#### 3.2 Proposed Genetic Algorithm structure

##### Chromosome and a fitness value

Since the objective function (6) is the function of  $S_i$  variables, each chromosome of the GA must be encoded with genes representing values of the order-up-to levels ( $S_i$ ). Fitness value of an individual chromosome can be evaluated by solving a set of problem given in (6) for a set of specified values  $S_i$  in the chromosome. The average of LP optimal solutions,  $E_D[Z^*]$ , is the fitness value of a chromosome. Figure 3 depicted the chromosome encoding and the evaluation of fitness value where a total of  $N_d$  demand patterns are evaluated.

$S_1$ $S_2$ . . . $S_n$	Avg. Fitn. Value	Fitness Value for Each Demand Pattern			
		$D1$	$d2$	. . .	$d_{Nd}$
15 20 . . . 15	28.3	25.4	37.2	. . .	19.8

Figure 3: Chromosome Encoding and The Evaluation of Fitness Value

##### Genetic Operators

For crossover operations generating offspring, we use a convex crossover operator defined by  $S_c = \alpha S_{p1} + (1-\alpha)S_{p2}$ , where  $S_c$  is a new value of  $S_i$  in an offspring,  $S_{p1}$  and  $S_{p2}$  are the current values in parents, and  $\alpha$  is a random weight ranged over  $0 \leq \alpha \leq 1$ . This convex operator takes a weighted average of the  $S_i$  values of the parents, and produces an offspring that lies somewhere on the line between the parents.

##### Genetic Algorithm Procedure

We have already designed the solution approach including 2 main procedures of the LP and the GA. The integrated optimization process is stated in a procedural form follows:

Step 1. Randomly generate an initial population containing  $n$  chromosomes of  $S = \{S_i\}$ .

Step 2. Set the population size,  $psize = n$ , set  $N_d$  = the number of generated demand patterns, for each chromosome in the population, generate  $N_d$  demand values as  $D_{ij} = \{d_{i,Nd}\}$ ,  $d_{ij} = \mu_i + \text{randn} * \sigma_i$ , where  $j = 1, 2, 3, \dots, Nd$ , and  $\text{randn}$  = a random number of  $Z \sim (0, 1)$ . Note that we have an alternative to maintain the same set

of generated demand values by resetting a seed number of standard normal random numbers  $Z$ .

Step 3. Executing a linear programming routine for the minimization of  $\text{Min}_X Z(X|S,D)$  problem, for given sets of  $\{S\}$  and  $\{D\}$  associated with each chromosome. Repeating the LP routine for all  $d_{i,Nd}$ , Record  $\{Z^*_{Nd}\}$  for the specified chromosome.

Step 4. Finding an expected value ( $E_D$ ) for a specified chromosome,  $E_D = (\sum Z^*_{Nd})/Nd$ . The expected value here is equivalent to a fitness value of a chromosome, and is representing the expected total cost of the system operating with the settings of  $\{S\}$  policies under the given  $\{D\}$ .

Step 5. Evaluate the remaining chromosomes by repeating step 2-4 throughout  $psize$

After the fitness evaluation process is done for any population  $psize$ , we can invoke the Genetic Algorithm routine for the minimization of  $\text{Min}_S E_D[Z^*(X^* |S,D)]$  problem as,

Step 6. Returning a set of fitness values  $\{E_D\}$  of  $n$  chromosomes to the GA routine.

Step 7. Record the best fitness value  $E_D^*$  and solutions  $\{S_i^*\}$  for the current population.

Step 8. If the stopping condition is met, then the solution of  $\text{Min}_S E_D[Z^*(X^* |S,D)]$  problem are  $E_D^*$  and  $\{S_i^*\}$ , End the optimization process. Otherwise, go to step 9.

Step 9. Set elite offsprings to survive.

Step 10. Perform a *Roulette Wheel Selection* [12].

Step 11. Set  $\alpha = \text{rand}[0,1]$  (or a predetermined value), and operate a convex genetic operator as given by a function,  $S_c = \alpha S_{p1} + (1-\alpha)S_{p2}$ , repeat the crossover process to produce a set of offspring which contain  $\{S_{c,i}\}$ .

Step 12. Perform a mutation process to completely fill a new population.

Step 13. Restart the Genetic Algorithm solution procedure by going to Step 2.

## 4. Experiment Results

To show the validity of the model, the solution approach, and the quality of the solutions  $E_D^*$  and  $\{S_i^*\}$ , we have solved 4 different hypothetical experiments with different settings of parameters. Table 1 depicts the parameter settings for experiments.

**Table 1:** Parameter Settings for The Experiments Conducted

Locations			Demand Characteristics		Parameter Settings			
DC	Art.DC	Retailers	Distribution	$\sigma/\mu$	Emergency Order	Lateral Transshipment	$h_i$	$p_i$
1	N/A	4	Normal	0.3	N/A	equal costs, $c_{ij}=10$	1	50
1	1	4	Normal	0.3	equal costs, $c_{0j}=20$	equal costs, $c_{ij}=10$	1	50
1	1	4	Normal	0.4	equal costs, $c_{0j}=20$	equal costs, $c_{ij}=10$	1	50
1	1	4	Normal	0.4	different costs $c_{0j} = \text{range } [20,50]$	different costs $c_{ij} = \text{range } [10,50]$	1	50

The genetic algorithm model is programmed in Matlab with the use of some built-in functions of Genetic Algorithm toolbox. The following criterion is used while running the model for validation:

1) Use the same set of average demand values for all experiments. The variance of demand patterns is changed for the last two experiments as we focus on the respond of the model to the changes in demands.

2) Neglect some part of a Coarse-To-Fine approach. The population size is not varied to reduce the computational time but we expedited the optimization process by controlling the diversity of chromosomes in the initial population. The range of initial population is restricted to be close to the average of demands.

3) The algorithm stops when the value of the fitness function for the best point in the current population is converged closely to the average of the fitness values and stall for some consecutive generations.

Experiment (1) is a base model without the integrated emergency ordering system, which will be used as a benchmark. This experiment case represents the multi-location inventory system with lateral transshipment that is similar to the models in [5], [6]. Holding costs ( $h_i$ ) and shortage penalty costs ( $p_i$ ) are set to 1 and 50, respectively to reflect that the shortages are expensive and product shipment is preferred. The same lateral transshipment costs and emergency ordering cost are used in the first

three experiments so that the changes in the expected total cost and policies  $\{S\}$  can be easily observed. The realization of demand in experiment (3) and (4) is made stochastically by varying the ratio of  $\sigma/\mu$  so that these experiments can represent the integrated system under different conditions. Finally, in experiment (4), the purchasing costs for lateral transshipments and emergency orders are given as random values within in the predetermined ranges.

The results for the 4 experiments are summarized in table 2, which gives the final values of  $E_D^*$  and  $S_i^*$ , that are rounded to the closet integers.

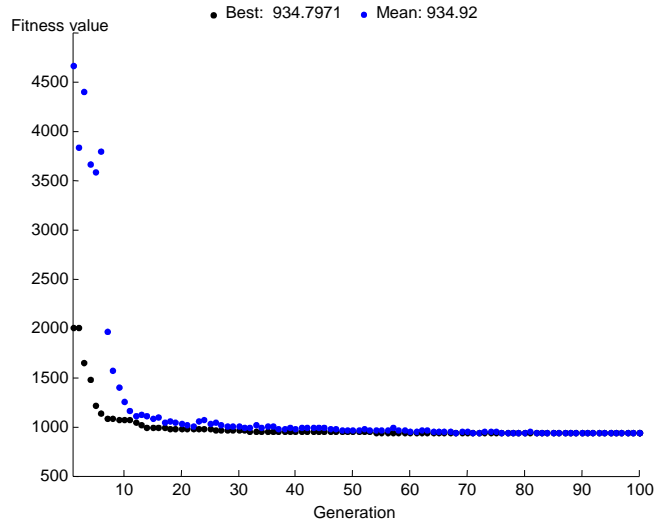
**Table 2:** Experiment Results

Exp.	Expected Cost (average demand)	Order-Up-To policies $\{S\}$			
		S1	S2	S3	S4
		(250)	(350)	(150)	(550)
1	721	364	522	213	704
2	588	336	528	208	695
3	779	353	547	227	720
4	934	403	574	254	934

With the presence of the emergency ordering system, we can expect that the results from our model should show the reduction in the total cost as well as the improvement in the order-up-to levels. Cost comparison of experiment case (1) and (2) shows that both the overall expected cost and the order-up-to levels at retailers decrease. The results match with the conclusion by Tagaras [7] that the emergency replenishment can improve the performance of a periodic review inventory system. If we focus on the respond of our model to any changes in the demand pattern, the results of experiment (3) indicates that the model could determine the optimal stocking levels  $\{S^*\}$  which are increased as the uncertainty in demands increases. The result complies with the inventory practice that the retailers require more stock when the demand is fluctuated. The expected total cost is also higher when extra shipments are preferred. This can be explained by the expression  $\sum_{ij} c_{ij} \cdot x_{ij}$  in the objective function (1) that stands for additional cost when the emergency order is transferred from the artificial retailer ( $i = 0$ ). When comparing the same results with the benchmark case (1), it can be claimed that the network with integrated emergency ordering system could perform better with similar settings of  $\{S\}$  under high demand uncertainty. It is shown here that the model could adjust the optimal values of  $\{S^*\}$  and  $E_D^*$  for the proposed system under different operating conditions. The parameter settings of experiment (4) are set to reflect the real-life situation that the stocking locations operate under different demand and cost parameters. As we integrated two alternative shipment systems, the model would allocate the additional

supply based on the retailer operating costs and the availability of product. The results of experiment (4) shows a higher optimal cost ( $E_D^*$ ), and the model adjusted new optimal replenishment levels of inventory  $\{S^*\}$  for all retailers based on the values of  $\{C_{ij}\}$  and demand patterns.

Finally, we illustrate clearly the convergence of our genetic algorithm for the experiment (4) in Figure 4. Note that, a set of dots that forms the bottom line in the plot represents the best expected cost value ( $E_D$ ) of chromosomes in each generation.



**Figure 4:** The Convergence of The Genetic Algorithm Optimization Process

## 5. Conclusion

In this paper, we present a multi-location inventory system where both lateral transshipments and emergency replenishments are allowed. We first investigate the possibility to consider the emergency replenishment in the inventory control decision, and then formulate a mathematical model for the determination of the optimal order-up-to replenishment policies in such system. We propose the solution approach which is the combination of two optimization procedures: a genetic algorithm model and a linear programming sub-model. It is found that the optimal solutions can be easily determined under the stochastic behavior. The future research based on our multi-location inventory model would be the incorporation of capacity constraints for a capacitated system, and the extension to the network design problem.

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