

# INTEGRATION OF JOB SCHEDULING WITH JOB DELIVERIES TO TWO CUSTOMER AREAS

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## ABSTRACT

In this study, a single machine scheduling model was developed that incorporated delivery vehicle routing decisions which serves two customer areas. The objective is to minimize the mean arrival time. The problem is NP-hard in the strong sense in general. The integer programming model was developed to optimally solve this problem. Also, computational results of the illustrative example are reported using the integer programming model to solve the proposed problem.

## KEY WORDS

Scheduling, single machine, integer programming, arrival time

## 1. Introduction

The coordination of activities along different stages of a supply chain has received much attention in production and operations management research. The tendency has created closer interactions between the stages in a supply chain increasing the usefulness of integrated models. In such an integrated system, the linkage between job scheduling (the production stage) and finished goods delivery (the distribution stage) is very important [1]. Traditional approaches separately and sequentially consider machine scheduling and job delivery, without effective coordination between the two. However, making two individual but uncoordinated decisions will not necessarily produce a global optimal solution. Substantial ineffectiveness may result when decision-making between the two stages is poorly coordinated, particularly if transportation is scarce in the system.

Lee and Chen [2] investigated machine scheduling problems with precise transportation considerations. Two types of transportation situations are considered in their

paper. Type-1 is transportation of jobs from one machine to another for further processing. Type-2 is the transportation provided to delivery of finished jobs to their destinations. The following study concentrates on the delivery of finished jobs to customer(s) (Type-2 transportation as defined by Lee and Chen [2]).

Problems in which jobs are delivered to customers in batches have seldom been addressed. Among the several survey papers on scheduling and batching problems include those by Potts and Van Wassenhove [3], Webster and Baker [4], Potts and Kovalyov [5], and Hall and Potts [6]. Herrmann and Lee [7], Chen [8], Cheng *et al.* [9], Yuan [10], Equi *et al.* [11], and Wang and Cheng [12], Garcia *et al.* [13] considered machine scheduling problems with jobs delivered in batches after being processed. Each delivery batch incurred a certain transportation cost. However, the above authors did not consider transportation times, that is, instantaneous deliveries were assumed.

Lee and Chen [2] have incorporated transportation time and vehicle capacity requirements into machine scheduling models. They analyzed the complexity of various models in which jobs were first processed by machines then delivered by one or more vehicles to a single customer. The vehicles have finite or infinite capacity and there is a transportation time for each direction of the delivery. Chang and Lee [1] have extended Lee and Chen's [2] work to the situation when each job occupies a different amount of space in the vehicle. They discussed three scenarios of the problem, including: (1) jobs which are processed on a single machine and delivered by a single vehicle to one customer area; (2) jobs which are processed by either one of two parallel machines and delivered by a single vehicle to one customer area, and (3) jobs to be processed by a single machine and delivered by a single vehicle to two

customer areas. Li *et al.* [14] developed a single machine scheduling model incorporating routing decisions of a delivery vehicle serving multiple customers at different locations. Except for multiple customer locations Li *et al.* [14] set the sum of job arrival times as the objective.

Chang and Lee [1] studied, for the first time, problems in which each job might occupy differing amounts of physical space in a transport vehicle. Based on machine scheduling and finished product delivery as discussed by Chang and Lee [1], this study deals with the situation in which jobs are processed on a single machine and delivered by a single vehicle to two customer areas. The objective is to determine the job processing sequence in the shop together with the delivery schedule to minimize the mean arrival time. This study presents an integer programming models for optimal mean arrival time. Also, computational results of the illustrative example are reported using the integer programming model to solve the proposed problem.

## 2. Problem Description and Notation Definition

This study investigates a class of the two-stage scheduling problem where the first stage is job production and the second is job delivery. The investigative focus is on integrating production scheduling with delivery of finished products to two customer areas. In this problem, jobs are first processed in a single machine then delivered in batches by a vehicle to multiple customer areas. Jobs require varying physical space while being loaded into a vehicle and delivered to a single customer. The vehicle is associated with a capacity constraint, measured by the total physical space of the jobs it can deliver in one trip and has a specific transportation time for each delivery. Job completion time denotes the time when a job arrives at the customer. This study aimed to minimize mean arrival times of all jobs to the customer.

The proposed problem was described as follows. A set of  $n$  jobs,  $N = \{J_1, J_2, \dots, J_n\}$ , had first to be processed in a manufacturing system by a single machine and then delivered to two customer areas. Each job,  $J_i$ , needed a processing time of  $p_i$  in the manufacturing system. Let  $e_i$  be the size of  $J_i$ , representing the physical space  $J_i$  occupied when loaded in the vehicle. One vehicle was available for delivery, with a capacity of  $z$  and was first located at the manufacturing facility. Vehicle capacity was measured by the total physical space the vehicle provides for one delivery. Assuming that while the total physical space of jobs loaded did not exceed  $z$ , they could be arranged to fit in the physical space provided by the vehicle. A delivery batch denotes all jobs delivered

together in one shipment and a transportation time is associated with each delivery batch. Furthermore, we define two customer areas as two locations. In this study, the situation of deliveries made to two customer areas was considered. Let  $t_{jj'}$  be the two-way travel time from customer area  $j$  to customer area  $j'$ , where  $j, j' = 0, 1, 2$ . Specifically, the location of single machine is called area 0.

The following notation was used throughout the study:

- $J_i$  = job number  $i$ ;
- $B_k$  = the  $k$ th delivery batch;
- $H$  = a very large positive number;
- $n$  = number of jobs for processing at time zero;
- $z$  = the vehicle capacity, that is, the total physical space provided by the vehicle for one delivery;
- $e_i$  = the physical space  $J_i$  occupies when loaded in the vehicle;
- $p_i$  = the processing time of  $J_i$ ;
- $Q_s$  = the number of route types which belong delivery type  $s$ .  $Q_1 = Q_2 = 3$  and  $Q_3 = 6$ .
- $t_{jj'}$  = the travel time from the area  $j$  to the area  $j'$ ;
- $u_{ij}$  = 1 if  $J_i$  must be delivered to area  $j$ ; 0 otherwise;
- $A_i$  = the arrival time of  $J_i$ , that is, the time when the vehicle finished delivering  $J_i$  to the customer area;
- $d_k$  = the departure time from the machine for the vehicle to deliver  $B_k$ ;
- $r_k$  = the ready time of  $B_k$ , representing the latest completion time on the machine of the jobs assigned to  $B_k$  on the machine. Note  $u_k \geq r_k$  in any feasible solution;
- $V_{kq}^s$  = 1 if route type  $q$  belongs delivery category  $s$  at batch  $B_k$ ; 0 otherwise;
- $w_k$  = 1 if batch  $B_k$  is not null batch; 0 otherwise;
- $W_{kj}$  = 1 if batch  $B_k$  must be delivered customer area  $j$ ; 0 otherwise;
- $Y_{ik}$  = 1 if  $J_i$  was scheduled at batch  $k$ ; 0 otherwise;

## 3. The Optimization Model

The optimization model employed integer programming technique to find the optimal solution for the proposed problem.

The objective was found in Eq. (1). The objective was to minimize the average arrival time of the set of orders during the time horizon.

$$\text{Minimize } \frac{1}{n} \sum_{i=1}^n A_i \quad (1)$$

Constraint set (2) ensured that each order could be processed on only one batch.

$$\sum_{k=1}^n Y_{ik} = 1 \quad i = 1, 2, \dots, n \quad (2)$$

Constraint set (3) ensured that if  $\sum_{i=1}^n Y_{ik} = 0$  then  $\sum_{i=1}^n Y_{i,k+1}$  could not equal 1 for  $k = 1, 2, \dots, n - 1$ . That is, if no orders placed at batch number  $k$  then also no orders placed at batch number  $k + 1$ .

$$\sum_{i=1}^n Y_{i,k+1} \leq H \sum_{i=1}^n Y_{ik} \quad k = 1, 2, \dots, n - 1 \quad (3)$$

Constraint set (4) restricted the total batch size to vehicle capacity.

$$\sum_{i=1}^n e_i Y_{ik} \leq z \quad k = 1, 2, \dots, n \quad (4)$$

We introduce binary variable  $w_k$  to describe whether that batch  $B_k$  is null batch or not. Constraint set (5) restricted  $w_k = 1$  if batch  $B_k$  is not null batch, while Constraint set (6) restricted  $w_k = 0$  if batch  $B_k$  is null batch.

$$\sum_{i=1}^n Y_{ik} \leq H \times w_k \quad k = 1, 2, \dots, n \quad (5)$$

$$w_k \leq H \sum_{i=1}^n Y_{ik} \quad k = 1, 2, \dots, n \quad (6)$$

Constraint sets (7) states the ready time of batch  $B_k$ , if  $w_k = 1$  then  $r_k \geq \sum_{k'=1}^k \sum_{i=1}^n p_i Y_{ik'}$ .

$$r_k \geq \sum_{k'=1}^k \sum_{i=1}^n p_i Y_{ik'} - H(1 - w_k) \quad k = 1, 2, \dots, n \quad (7)$$

Constraint sets (8) and (9) define binary variable  $W_{kj}$ . If batch  $B_k$  must be delivered to customer area 1, then  $W_{k1} = 1$ . Otherwise  $W_{k1} = 0$ . That is,  $\sum_{i=1}^n u_{i1} Y_{ik} \leq H \times W_{k1}$  and  $W_{k1} \leq H \sum_{i=1}^n u_{i1} Y_{ik}$ . Similarly, If batch  $B_k$  must be delivered to customer area 2, then  $W_{k2} = 1$ . Otherwise  $W_{k2} = 0$ . That is,  $\sum_{i=1}^n u_{i2} Y_{ik} \leq H \times W_{k2}$  and  $W_{k2} \leq H \sum_{i=1}^n u_{i2} Y_{ik}$ .

$$\sum_{i=1}^n u_{ij} Y_{ik} \leq H \times W_{kj} \quad k = 1, 2, \dots, n; j = 1, 2 \quad (8)$$

$$W_{kj} \leq H \sum_{i=1}^n u_{ij} Y_{ik} \quad k = 1, 2, \dots, n; j = 1, 2 \quad (9)$$

If batch  $B_k$  is not a null batch, then there has job(s) must be delivered to one or two customer areas. There are three delivery categories to the customer areas, including deliver only to area 1, deliver only to area 2, and deliver to both areas 1 and 2. Figure 1 illustrates that there are three delivery types for delivering only to area 1, that is,  $Q_1 = 3$ . Figure 2 illustrates that there are three delivery types for delivering only to area 2, that is,  $Q_2 = 3$ . Figure 3 illustrates that there are six delivery types for delivering to both areas 1 and 2, that is,  $Q_3 = 6$ .

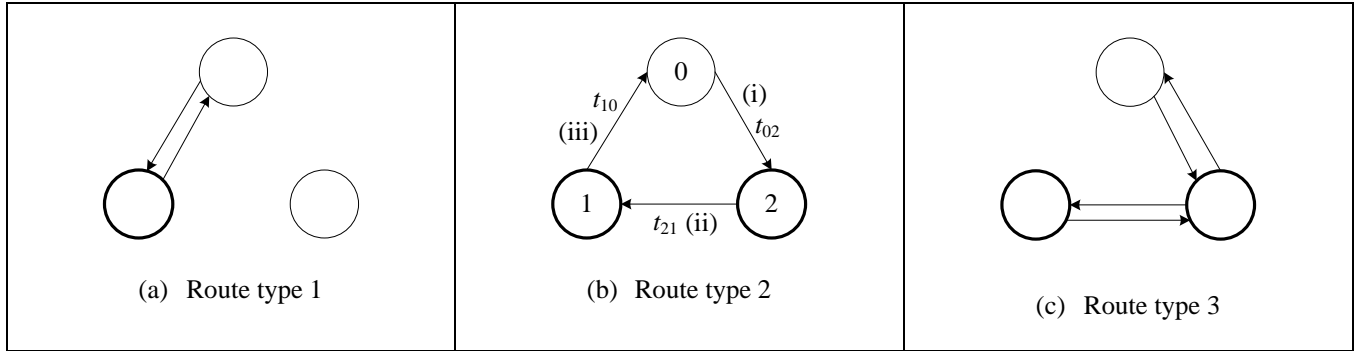


Figure 1: Illustration of The Three Route Types for Delivering Only to Area 1

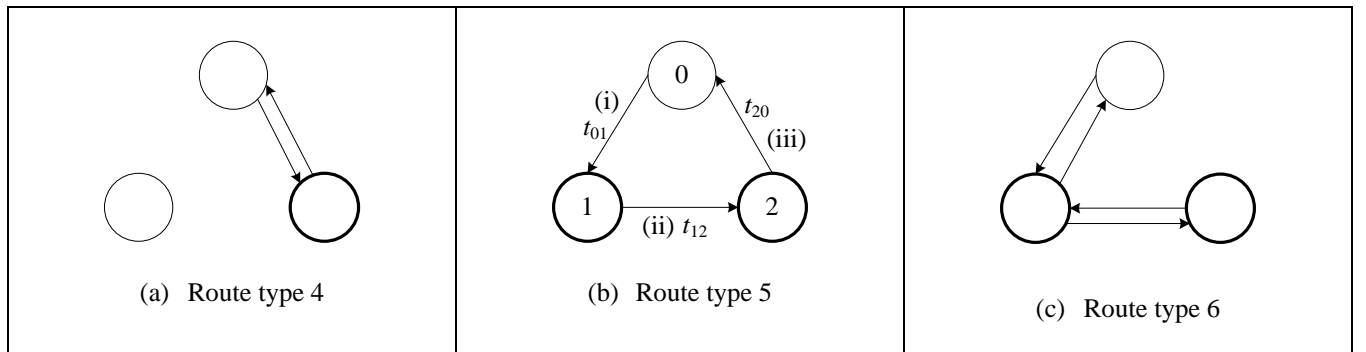
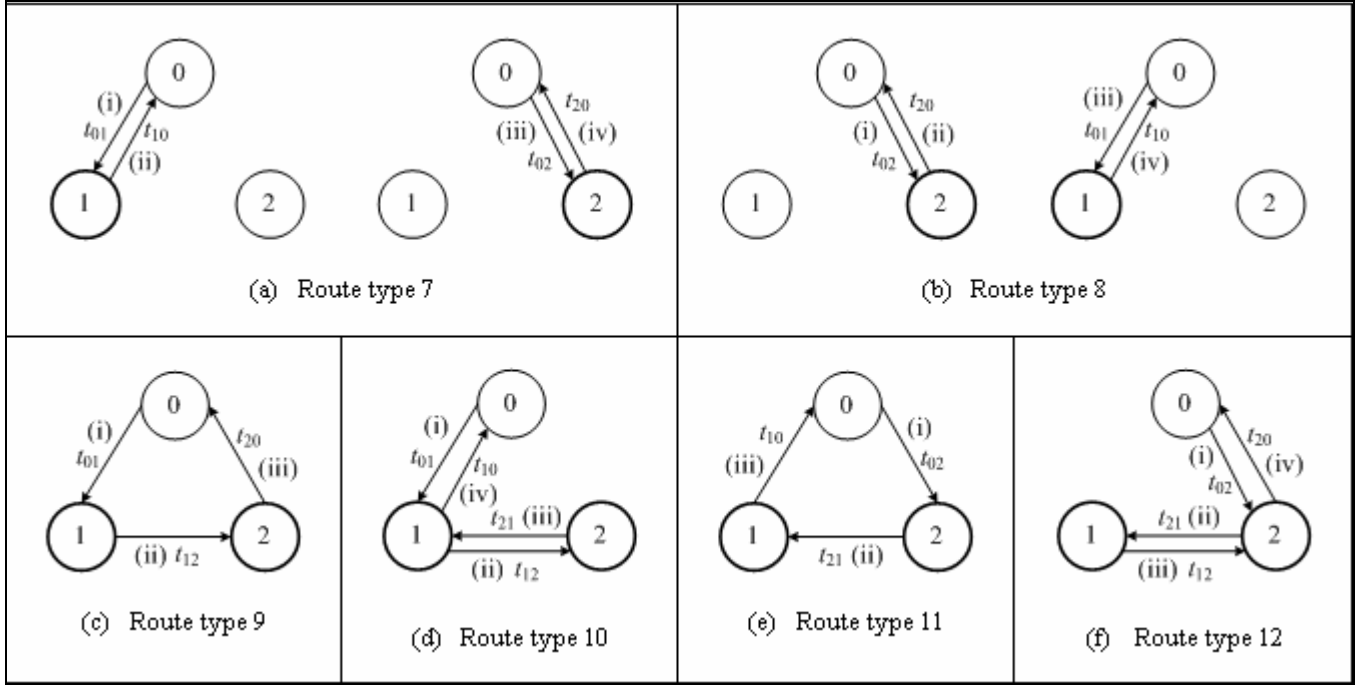


Figure 2: Illustration of The Three Route Types for Delivering Only to Area 2



**Figure 3:** Illustration of The Six Route Types for Delivering to Areas 1 and 2

We introduce binary variable  $V_{kq}^s$  to describe whether route type  $q$  belongs delivery category  $s$  at batch  $B_k$  or not. If batch  $B_k$  is a null batch, then there are not any jobs must be delivered to any of customer areas. That is, if  $w_k = 0$ , then  $\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s = 0$ .

$$\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s \leq H \times w_k \quad k = 1, 2, \dots, n \quad (10)$$

If batch  $B_k$  is not a null batch, then the route type is just only one and belongs to the twelve possible delivery types. That is, if  $w_k = 1$ , then  $\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s = 1$ . If  $w_k = 0$ , then  $\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s = 0$ .

$$\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s \leq 1 \quad k = 1, 2, \dots, n \quad (11)$$

Constraint sets (12)-(15) describe the relations between  $W_{kj}$  and  $V_{kq}^s$ . In constraint set (12), if  $W_{k1} = W_{k2} = 0$  then  $\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s = 0$ . In constraint set (13), if  $W_{k1} = 1$  and  $W_{k2} = 0$  then  $\sum_{q=1}^3 V_{kq}^1 = 1$ . In constraint set (14), If  $W_{k1} = 0$  and  $W_{k2} = 1$  then  $\sum_{q=1}^3 V_{kq}^2 = 1$ . In constraint set (15), If  $W_{k1} = 1$  and  $W_{k2} = 1$  then  $\sum_{q=1}^6 V_{kq}^3 = 1$ .

$$\sum_{s=1}^3 \sum_{q=1}^{Q_k} V_{kq}^s \leq H(W_{k1} + W_{k2}) \quad k = 1, 2, \dots, n \quad (12)$$

$$\sum_{q=1}^3 V_{kq}^1 \geq 1 - H(1 - W_{k1} + W_{k2}) \quad k = 1, 2, \dots, n \quad (13)$$

$$\sum_{q=1}^3 V_{kq}^2 \geq 1 - H(1 - W_{k2} + W_{k1}) \quad k = 1, 2, \dots, n \quad (14)$$

$$\sum_{q=1}^6 V_{kq}^3 \geq 1 - H(2 - W_{k1} - W_{k2}) \quad k = 1, 2, \dots, n \quad (15)$$

The relations between the ready time and the departure time for each batch were defined by constraint sets (16) and (17).

$$d_1 = r_1 \quad (16)$$

$$d_k \geq r_k \quad k = 2, 3, \dots, n \quad (17)$$

Constraint sets (18)-(29) are the departure time restrictions for three delivery categories to the customer areas, respectively. Constraint sets (18)-(20) are the departure time restrictions for delivering only to area 1. Constraint sets (21)-(23) are the departure time restrictions for delivering only to area 2. Constraint sets (24)-(29) are the departure time restrictions for delivering to both areas 1 and 2.

$$d_k \geq d_{k-1} + t_{01} + t_{10} - H(2 - V_{k-1,1}^1 - w_k) \quad k = 2, 3, \dots, n \quad (18)$$

$$d_k \geq d_{k-1} + t_{02} + t_{21} + t_{10} - H(2 - V_{k-1,2}^1 - w_k) \quad k = 2, 3, \dots, n \quad (19)$$

$$d_k \geq d_{k-1} + t_{02} + t_{21} + t_{12} + t_{20} - H(2 - V_{k-1,3}^1 - w_k) \quad k = 2, 3, \dots, n \quad (20)$$

$$d_k \geq d_{k-1} + t_{02} + t_{20} - H(2 - V_{k-1,1}^2 - w_k) \quad k = 2, 3, \dots, n \quad (21)$$

$$d_k \geq d_{k-1} + t_{01} + t_{12} + t_{20} - H(2 - V_{k-1,2}^2 - w_k) \quad k = 2, 3, \dots, n \quad (22)$$

$$d_k \geq d_{k-1} + t_{01} + t_{12} + t_{21} + t_{10} - H(2 - V_{k-1,3}^2 - w_k) \quad k = 2, 3, \dots, n \quad (23)$$

$$d_k \geq d_{k-1} + t_{01} + t_{10} + t_{02} + t_{20} - H(2 - V_{k-1,1}^3 - w_k) \quad k = 2, 3, \dots, n \quad (24)$$

$$d_k \geq d_{k-1} + t_{01} + t_{10} + t_{02} + t_{20} - H(2 - V_{k-1,2}^3 - w_k) \quad k = 2, 3, \dots, n \quad (25)$$

$$d_k \geq d_{k-1} + t_{01} + t_{12} + t_{20} - H(2 - V_{k-1,3}^3 - w_k) \quad k = 2, 3, \dots, n \quad (26)$$

$$d_k \geq d_{k-1} + t_{01} + t_{12} + t_{21} + t_{10} - H(2 - V_{k-1,4}^3 - w_k) \quad k = 2, 3, \dots, n \quad (27)$$

$$d_k \geq d_{k-1} + t_{02} + t_{21} + t_{10} - H(2 - V_{k-1,5}^3 - w_k) \quad k = 2, 3, \dots, n \quad (28)$$

$$d_k \geq d_{k-1} + t_{02} + t_{21} + t_{12} + t_{20} - H(2 - V_{k-1,6}^3 - w_k) \quad k = 2, 3, \dots, n \quad (29)$$

Constraint sets (30) and (47) define the arrival time  $A_i$  for three delivery categories to the customer areas, respectively. Constraint sets (30)-(32) define the arrival time  $A_i$  for delivering only to area 1. Constraint sets (33)-(35) define the arrival time  $A_i$  for delivering only to area 2. Constraint sets (36)-(47) define the arrival time  $A_i$  for delivering to both areas 1 and 2.

$$A_i \geq d_k + t_{01} - H(2 - u_{i1}Y_{ik} - V_{k1}^1) \quad i, k = 1, 2, \dots, n \quad (30)$$

$$A_i \geq d_k + t_{02} + t_{21} - H(2 - u_{i1}Y_{ik} - V_{k2}^1) \quad i, k = 1, 2, \dots, n \quad (31)$$

$$A_i \geq d_k + t_{02} + t_{21} - H(2 - u_{i1}Y_{ik} - V_{k3}^1) \quad i, k = 1, 2, \dots, n \quad (32)$$

$$A_i \geq d_k + t_{02} - H(2 - u_{i2}Y_{ik} - V_{k1}^2) \quad i, k = 1, 2, \dots, n \quad (33)$$

$$A_i \geq d_k + t_{01} + t_{12} - H(2 - u_{i2}Y_{ik} - V_{k2}^2) \quad i, k = 1, 2, \dots, n \quad (34)$$

$$A_i \geq d_k + t_{01} + t_{12} - H(2 - u_{i2}Y_{ik} - V_{k3}^2) \quad i, k = 1, 2, \dots, n \quad (35)$$

$$A_i \geq d_k + t_{01} - H(2 - u_{i1}Y_{ik} - V_{k1}^3) \quad i, k = 1, 2, \dots, n \quad (36)$$

$$A_i \geq d_k + t_{01} + t_{10} + t_{02} - H(2 - u_{i2}Y_{ik} - V_{k1}^3) \quad i, k = 1, 2, \dots, n \quad (37)$$

$$A_i \geq d_k + t_{02} + t_{20} + t_{01} - H(2 - u_{i1}Y_{ik} - V_{k2}^3) \quad i, k = 1, 2, \dots, n \quad (38)$$

$$A_i \geq d_k + t_{02} - H(2 - u_{i2}Y_{ik} - V_{k2}^3) \quad i, k = 1, 2, \dots, n \quad (39)$$

$$A_i \geq d_k + t_{01} - H(2 - u_{i1}Y_{ik} - V_{k3}^3) \quad i, k = 1, 2, \dots, n \quad (40)$$

$$A_i \geq d_k + t_{01} + t_{12} - H(2 - u_{i2}Y_{ik} - V_{k3}^3) \quad i, k = 1, 2, \dots, n \quad (41)$$

$$A_i \geq d_k + t_{01} - H(2 - u_{i1}Y_{ik} - V_{k4}^3) \quad i, k = 1, 2, \dots, n \quad (42)$$

$$A_i \geq d_k + t_{01} + t_{12} - H(2 - u_{i2}Y_{ik} - V_{k4}^3) \quad i, k = 1, 2, \dots, n \quad (43)$$

$$A_i \geq d_k + t_{02} + t_{21} - H(2 - u_{i1}Y_{ik} - V_{k5}^3) \quad i, k = 1, 2, \dots, n \quad (44)$$

$$A_i \geq d_k + t_{02} - H(2 - u_{i2}Y_{ik} - V_{k5}^3) \quad i, k = 1, 2, \dots, n \quad (45)$$

$$A_i \geq d_k + t_{02} + t_{21} - H(2 - u_{i1}Y_{ik} - V_{k6}^3) \quad i, k = 1, 2, \dots, n \quad (46)$$

$$A_i \geq d_k + t_{02} - H(2 - u_{i2}Y_{ik} - V_{k6}^3) \quad i, k = 1, 2, \dots, n \quad (47)$$

Finally, constraint set (48) specifies the non-negativity of  $A_i$ ,  $r_k$ , and  $d_k$ , and establishes the binary restrictions for  $w_k$ ,  $W_{kj}$ ,  $Y_{ik}$ , and  $V_{kq}^s$ .

$$\begin{aligned} A_i &\geq 0 \quad i = 1, 2, \dots, n; \quad r_k \geq 0, \quad d_k \geq 0 \quad k = 1, 2, \dots, n; \\ w_k &\text{ is binary} \quad k = 1, 2, \dots, n; \\ W_{kj} &\text{ is binary} \quad k = 1, 2, \dots, n; \quad j = 1, 2; \\ Y_{ik} &\text{ is binary} \quad i, k = 1, 2, \dots, n; \\ V_{kq}^s &\text{ is binary} \quad k = 1, 2, \dots, n; \quad q = 1, 2, \dots, Q_s; \\ &\quad s = 1, 2, 3 \end{aligned} \quad (48)$$

#### 4. An Illustrative Example

The proposed integer programming model is illustrated here. Six jobs ( $n = 6$ ) are to be scheduled, namely  $J_1$  to  $J_6$ . In this example, there are six jobs, one machine, two customer areas and one vehicle with a capacity of 10 ( $z = 10$ ). Processing time is  $p_1 = 2, p_2 = 1, p_3 = 5, p_4 = 2, p_5 = 6, p_6 = 5$ . The physical space is  $e_1 = 5, e_2 = 2, e_3 = 4, e_4 = 8, e_5 = 2, e_6 = 6$ . The number of route types are  $Q_1 = 3, Q_2 = 3, Q_3 = 6$ . The two-way travel time between machine and customer area is as follows.

$$t_{j'j''} = \begin{bmatrix} t_{00} & t_{01} & t_{02} \\ t_{10} & t_{11} & t_{12} \\ t_{20} & t_{21} & t_{22} \end{bmatrix} = \begin{bmatrix} \infty & 4 & 5 \\ 6 & \infty & 4 \\ 6 & 1 & \infty \end{bmatrix}$$

The binary variable  $u_{ij} = 1$  denote job  $J_i$  must be delivered to area  $j$ ;  $u_{ij} = 0$  otherwise. This example set the values of  $u_{ij}$  are as follows.

$$u_{ij} = \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \\ u_{31} & u_{32} \\ u_{41} & u_{42} \\ u_{51} & u_{52} \\ u_{61} & u_{62} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The problem is solved by the integer programming formulation and its optimal mean arrival time is 18.6667.

The values of related variables follow.  $Y_{11} = 1, Y_{21} = 1, Y_{32} = 1, Y_{43} = 1, Y_{53} = 1, Y_{62} = 1, B_1 = \{J_1, J_2\}, B_2 = \{J_3, J_6\}, B_3 = \{J_4, J_5\}, w_1 = 1, w_2 = 1, w_3 = 1, V_{11}^1 = 1, V_{25}^3 = 1, V_{35}^3$

$= 1, d_1 = 3, d_2 = 13, d_3 = 25, r_1 = 3, r_2 = 13, r_3 = 25, A_1 = 7, A_2 = 7, A_3 = 19, A_4 = 30, A_5 = 31, A_6 = 18$ . The related optimal schedule is shown in Fig. 4.

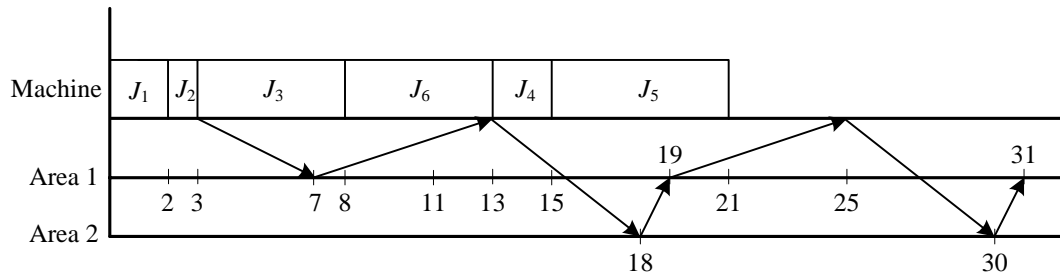


Figure 4: An Optimal Schedule for The Example

## 5. Conclusion and Future Research

This study considers single machine scheduling for jobs delivered to two customer areas. The objective was to minimize mean arrival time. An integer programming formulation was proposed to find the optimal schedule. Computational results of the illustrative example are reported using the integer programming model to solve the proposed problem. Future research should address problems with multiple customer areas or different shop environments, including flow-shop and job-shop. Problems with other performance measures, including minimum mean flow time, mean tardiness, and multi-criteria measures, should also be studied. Meta-heuristics could be used to achieve solutions.

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## References

[1] Y.C. Chang & C.Y. Lee, Machine scheduling with job delivery coordination, *European Journal of Operational Research*, 158, 2004, 470-487.  
 [2] C.Y. Lee & Z.L. Chen, Machine scheduling with transportation considerations, *Journal of Scheduling*, 4, 2001, 3-24.  
 [3] C.N. Potts & L.N. Van Wassenhove, Integrating scheduling with batching and lot-sizing: A review of algorithms and complexity, *Journal of the Operational Research Society*, 43, 1992, 395-406.  
 [4] S. Webster & K.R. Baker, Scheduling groups of jobs on a single machine, *Operations Research*, 43, 1995, 692-703.

[5] C.N. Potts & M.Y. Kovalyov, Scheduling with batching: A review, *European Journal of Operational Research*, 120, 2000, 228-249.  
 [6] N.G. Hall & C.N. Potts, Supply chain scheduling: Batching and delivery, *Operations Research*, 51, 2003, 566-584.  
 [7] J.W. Herrmann & C.Y. Lee, On scheduling to minimize earliness-tardiness and batch delivery costs with a common due date, *European Journal of Operational Research*, 70, 1993, 272-288.  
 [8] Z.L. Chen, Scheduling and common due date assignment with earliness-tardiness penalties and batch delivery costs, *European Journal of Operational Research*, 93, 1996, 49-60.  
 [9] T.C.E. Cheng, V.S. Gordon, & M.Y. Kovalyov, Single machine scheduling with batch deliveries, *European Journal of Operational Research*, 94, 1996, 277-283.  
 [10] J. Yuan, A note on the complexity of single-machine scheduling with a common due date, earliness-tardiness, and batch delivery costs, *European Journal of Operational Research*, 94, 1996, 203-205.  
 [11] L. Equi, G. Gallo, S. Marziale, & A. Weintraub, A combined transportation and scheduling problem, *European Journal of Operational Research*, 97, 1997, 94-104.  
 [12] G. Wang & T.C.E. Cheng, Parallel machine scheduling with batch delivery costs, *International Journal of Production Economics*, 68, 2000, 177-183.  
 [13] J.M. Garcia, S. Lozano, & D. Canca, Coordinated scheduling of production and delivery from multiple plants, *Robotics and Computer-Integrated Manufacturing*, 20, 2004, 191-198.  
 [14] C.L. Li, G. Vairaktarakis, & C.Y. Lee, Machine scheduling with deliveries to multiple customer locations, *European Journal of Operational Research*, 164(1), 2005, 39-51.